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#### Research paper

# NUMERICAL SIMULATIONS OF LABORATORY TESTS FOR TRIAXIAL APPARATUS

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#### **Abstract**

Numerical simulations of laboratory tests in triaxial apparatus are very important for modeling soil behavior in geotechnical earthquake engineering. Constitutive models in numerical simulations play essential role representing critical aspects of soil response under varying stress conditions and loading paths. Understanding the inherent limitations and applicability ranges of these models is crucial for selecting appropriate models for specific applications and interpreting results. This study presents comprehensive comparative analyses of several triaxial tests simulated using different constitutive material models, with soil modeled as a multiphase medium to better represent its true physical state. The triaxial monotonic drained and undrained tests have been performed at the Laboratory for Soil Dynamics and Foundation Engineering at the Institute of Earthquake Engineering and Engineering Seismology (IZIIS) in Skopje, N. Macedonia, in cooperation with colleagues from the Laboratory for Geotechnics of the Faculty of Civil Engineering and Architecture of Niš, Serbia. The collaborative nature of this research enhances the validity and reliability of the experimental results. The numerical analyses were performed using the finite element software PLAXIS, which offers a comprehensive suite of constitutive models, and the comparisons are discussed in detail.

Key words: Triaxial tests, numerical simulations, PLAXIS, finite element

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#### 1. INTRODUCTION

Constitutive models provide essential mathematical descriptions of the mechanical behavior of materials and are fundamental to representing the most important aspects of soil behavior in geotechnical engineering applications. These models serve as the mathematical framework that connects stress and strain relationships, facilitating the numerical simulation of complex soil response under various loading conditions. Despite significant advances in computational geomechanics over recent decades, no single material model exists that can successfully simulate all soil behaviors encountered in practice.

The limitations of elastic material models in soil behavior simulation are well-documented. Although computationally efficient and straightforward to implement, purely elastic models play a minimal role in accurately simulating soil behavior, because soils fundamentally do not behave as elastic materials except within an extremely limited strain range typically less than 0.001%. Beyond this threshold, soils exhibit complex non-linear, stress-path dependent behavior that requires more sophisticated modeling approaches.

The theory based on decomposition of strain into elastic and plastic components represents a powerful tool to describe the irreversible deformations characteristic of sandy structures. This elasto-plastic approach has become a cornerstone in geotechnical modeling, allowing engineers to predict not only failure conditions, but also pre-failure deformations that are often critical in design considerations. When the investigation is limited to failure mechanisms, a simple failure criterion such as the Mohr–Coulomb model is used [1].

For more comprehensive analysis requirements, strain hardening models [2-4] offer significantly more realistic displacement patterns than the basic Mohr–Coulomb criterion. The Hardening Soil model, a prominent example of advanced strain hardening models implemented in commercial software like PLAXIS, incorporates stress-dependent stiffness according to a power law and accounts for both shear and volumetric hardening. The strain hardening models incorporate the evolution of soil strength and stiffness with plastic straining, thereby capturing key aspects of soil behavior that simpler models neglect.

As generally observed by researchers, the inelastic behavior for sands is described within the yield surface framework, which provides a more realistic representation of soil response to loading. Hypoplastic models [5] represent another advanced class of constitutive relationships that capture realistically the influence of both mean pressure and relative density on soil behavior. These models offer substantial advantages in simulating the nonlinear response of granular materials without requiring an explicit yield surface definition. The nonlinear constitutive relation is particularly reliable when coupled with volumetric behavior calculations, including the critical effects of pore pressure development during loading. This capability makes hypoplastic models especially valuable for earthquake engineering applications where pore pressure evolution significantly influences soil strength and stiffness.

Recent advances in computational capabilities have enabled increasingly sophisticated implementations of these theoretical frameworks, allowing for more realistic simulations of soil behavior under both static and dynamic loading conditions. The selection of an appropriate constitutive model for a specific geotechnical problem remains a critical engineering decision that requires careful consideration of the model's capabilities, limitations, and parameter identification requirements. This comparative study aims to provide practical guidance for this selection process by systematically evaluating the

performance of different constitutive models in simulating triaxial test results under controlled laboratory conditions.

#### 2. SAMPLE PREPARATION IN TRIAXIAL APPARATUS

Cylindrical specimens of sandy soil, originating from the city of Skopje (hereinafter called Skopje sand), with dimensions of 140 mm in height and 70 mm in diameter were used for all test series, maintaining a height-to-diameter ratio (H/D) between 2 as recommended by standard for soil testing [6]. Due to the difficulty in preserving undisturbed samples of the investigated material and the requirement for numerous tests, all experiments were conducted on reconstituted specimens.

Various methods for reconstituting sand specimens are documented in the literature. Research has established that different reconstitution methods produce specimens with distinct structures and behavioral characteristics [7]. Therefore, it is crucial to consistently reproduce specimens with identical density and soil fabric to ensure reliable research outcomes. Previous studies have demonstrated that the method used to prepare sand to a specific density can significantly influence its settlement characteristics.

The Laboratory for Soil Dynamics and Foundation Engineering at the Institute of Earthquake Engineering and Engineering Seismology (IZIIS-Skopje) has extensive experience with the moist tamping technique for preparing samples of different materials. This technique has been successfully employed, as documented in the works of authors [8-12]. A key advantage of this method is the ability to prepare multiple samples at nearly identical densities.

All tests within this research framework were conducted on fully saturated samples for several reasons. First, soil typically exists below the groundwater level in field conditions and is therefore naturally saturated. Second, volume change measurements are considerably more straightforward in fully saturated samples during consolidation and drained cyclic loading compared to dry or partially saturated samples. In saturated conditions, volume changes can be directly measured by monitoring pore water entering or leaving the sample, whereas in dry or partially saturated conditions, volume changes are complicated by simultaneous changes in air volume within the soil voids.

The specimen is prepared by tamping the soil material in several layers considering the same density of soil layers (Figure 1). After the specimen preparation is finished, the triaxial cell top is placed onto the base. The ram clamp is used to support the ram, so that when the cell top is placed the ram does not make contact with the sample. The cell top is tightened by clamping screws and the ram is moved down towards the sample until it just touches the rubber suction ring. The actuator should be at about mid travel or just above. Using the load machine the triaxial cell is brought down to make contact with the sample. The displacement transducer fitted to the triaxial cell ram is arranged to be close to zero on the levels screen. Triaxial test consists of cylindrical soil specimen fenced within a rubber membrane inside a pressure cell. The lower and upper loading platens have porous disks connected to the drainage system for saturating and/or draining the soil specimen. The confining pressure is applied by adjusting the chamber pressure and the axial stress is applied by pushing the piston. The triaxial cell is filled with water, when water escapes from the air bleed close the cell valve to stop water entering. The system and specimen are ready to test as can be seen in Figure 2.



Figure 1. Triaxial apparatus - preparation of a sample for testing



Figure 2. Triaxial apparatus with soil specimen ready for testing

# 3. CONSTITUTIVE MODELING OF SOIL IN PLAXIS SOFTWARE

The PLAXIS finite element software incorporates several constitutive models of varying complexity to address challenges in geotechnical modeling. The software offers a range of models from simple linear elastic and Mohr–Coulomb models to more advanced formulations such as the Hardening Soil model and implementations of hypoplastic constitutive relationships. Each model represents different advantages between computational complexity, parameter requirements and capability to capture specific aspects of soil behavior. When selecting an appropriate constitutive model in PLAXIS for a particular geotechnical application, specific features of soil behavior are most relevant to the problem at hand. For problems involving small strains within a limited stress range, simpler models may provide sufficient accuracy.

A particular class of these models considering small deformations, which have received special attention in recent times, is provided by the theory of hypoplasticity. The material model of hypoplasticity defines the change of stress rate of the cohesionless granular materials as a function of initial stress  $\sigma$ , strain-rate  $\Delta\epsilon$  and initial void ratio e:

$$\Delta \sigma = f\left(\sigma, \Delta \varepsilon, e\right) \tag{1}$$

This formulation is of rate type which guarantees the load paths to be very similar to reality [13]. Since initial stress and density have impact on the overall deformation of the granular soils, this formulation has in it the stress state and the void ratio as input parameters. In this work the hypoplastic formulation according to Wolffersdorf [14] is used. In tensorial way it is written in equation (2) as

$$\Delta \sigma_{ii} = L_{iikl} \Delta \varepsilon_{kl} + N_{ii} \sqrt{\Delta \varepsilon_{kl} \Delta \varepsilon_{kl}}$$
 (2)

It is to be mentioned that for the simulation of hypoplastic modeling there are couple of parameters which should be defined at the laboratory. In this work, the parameters for the Skopje sand used can be summarized as follows:

Description of parameter	Value	
Angle of internal friction φc	35°	
Granulate hardness hs	2650 MPa	
Exponent n	0.26	
Minimum void ratio ed0	0.61	
Critical void ratio ec0	0.98	
Maximum void ratio ei0	1.09	
Numerical parameter α	0.07	
Numerical parameter β	2.0	

Table 1. Hypoplastic parameters of the tested Skopje sand

On the other hand, the models of Mohr–Coulomb and Hardening Soil make an important part in order to simulate the behavior of the soil. The Mohr–Coulomb model presented by equation (3), despite its simplicity, remains widely used in geotechnical practice and serves as a reference for more advanced constitutive models in PLAXIS:

$$\tau_f = c' + \sigma'_f \tan \varphi' \tag{3}$$

This model is characterized by a fixed yield surface defined by cohesion (c) and friction angle  $(\phi)$ , elastic behavior within the yield surface described by Young's modulus (E) and Poisson's ratio (v), and a non-associated flow rule governed by the dilatancy angle  $(\psi)$ .

The Hardening Soil model, on the other hand, represents a significant advancement over the basic Mohr–Coulomb formulation and was also implemented in this numerical simulations using PLAXIS. This advanced constitutive model incorporates stress-dependent stiffness according to a power law (controlled by parameter m), distinguishes between primary loading and unloading-reloading stiffness ( $E_{50}$ ,  $E_{oed}$  and  $E_{urr}$ ), and accounts for both shear and volumetric hardening. The determination of Hardening Soil parameters requires more comprehensive laboratory testing, including multiple triaxial tests at different confining pressures with precise strain measurements, particularly in the small strain range.

## 4. IMPLEMENTATION IN PLAXIS SOFTWARE

The implementation in the PLAXIS software is composed of remote scripting in which Python language is used to enable the input of the material parameters. The remote scripting functionality represents a significant advancement over traditional manual GUI-based model creation, allowing for parametric studies, automated verification and reproducible simulation workflows. The implementation process begins with establishing a connection to the PLAXIS remote scripting server as given in Figure 3.

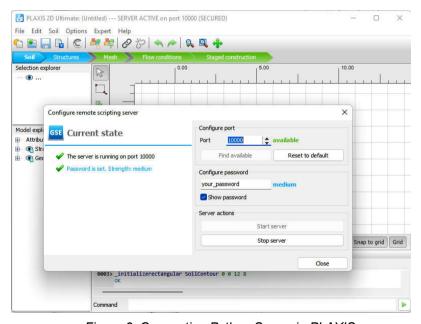


Figure 3. Connecting Python Server in PLAXIS

After the connection has been established, the PLAXIS kernel becomes accessible through Python commands, enabling programmatic control over all aspects of the simulation process. This connection provides a bidirectional communication channel between the Python environment and the PLAXIS computational engine as shown in Figure 4.

```
×
Jupyter QtConsole
File Edit View Kernel Window Help
Jupyter QtConsole 5.5.1
PLAXIS Interactive Python Console
Python 3.12.3 - IPython 8.23.0
Connected to localhost on port 10000
Available variables:
   s i: the application server
   g i: the global environment
Example session:
   >>> s_i.new()
   >>> g_i.soillayer(2)
In [1]: from plxscripting.easy import *
   ...: import matplotlib.pyplot as plt
  ...: # Connect to PLAXIS (ensure the app is running)
  ...: s i, g i = new server('localhost', 10000, password="your password")
   ...: s i.new()
Out[1]: 'OK'
In [2]:
```

Figure 4. The Python Console of the PLAXIS software

The main advantages of the interactive console over traditional GUI-based modeling include efficient parameter input, the ability to run multiple simulations with varying parameters and comprehensive documentation of all simulation inputs ensuring reproducibility. The input files for the material models are provided in the Appendix. As the file is processed, the PLAXIS software activates a specialized module for material testing, as given in the following Figure 5.

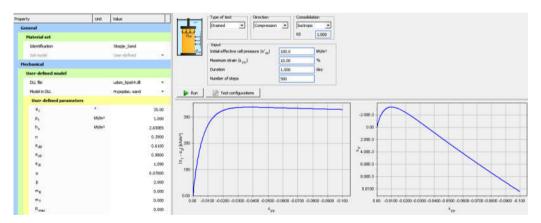


Figure 5. Testing module of PLAXIS software

# 5. ANALYSIS OF RESULTS

In analyzing the results first the **drained case of monotonic test** has been considered. During the test at the laboratory the valve has been left open in order to simulate the drained case of experiment. The drained triaxial test simulations provide instructive insights into the relative performance of the constitutive models. As illustrated in Figure 6, the experimental

stress–strain curves for Skopje sand exhibit characteristic non-linear behavior with distinct transitions from initially stiff response to progressively softer behavior as strains increase. The Mohr–Coulomb model, while correctly predicting the ultimate strength at large strains, significantly overestimates stiffness in the small to medium strain range. This results in unrealistic displacement predictions that would lead to substantial underestimation of settlements in practical applications. The model's linear elastic assumption prior to failure creates a bilinear stress–strain curve that fundamentally misrepresents the gradual yielding process occurring in the soil. In contrast, the Hardening Soil model demonstrates remarkable agreement with the experimental data across the entire strain spectrum. Its hyperbolic formulation successfully captures the progressive mobilization of shear strength and the continuous transition from small-strain elastic behavior to large-strain plastic deformation. Particularly noteworthy is the model's ability to reproduce both the initial tangent stiffness and the secant stiffness at 50% of failure stress ( $E_{50}$ ), which proves crucial for accurate settlement predictions. On the other hand, the Hypoplastic model results are closer to the experimental values due to the captured nonlinearity from the beginning of the strain development.

The comparative analysis clearly illustrates that while all models may converge to similar predictions at failure conditions, the deformation path to failure is substantially better represented by the Hardening Soil model and Hypoplastic model.

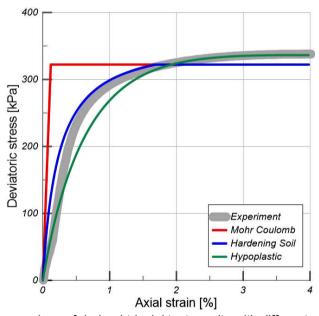


Figure 6. Comparison of drained triaxial test results with different material models

The **undrained triaxial test results**, as shown in Figure 7, reveal significant differences in how the models predict stress-strain behavior under undrained conditions. The experimental data (solid gray line) exhibits the characteristic non-linear response with initial stiffness at small strains gradually transitioning to a more plastic response at larger strains, ultimately approaching a plateau around 550 kPa deviatoric stress at 3% axial strain. The Mohr–Coulomb model together with the Hardening Soil and Hypoplastic model demonstrate good performance, closely tracking the experimental curve throughout most of the loading path. The main reason for the good correlation among the models is due to the fact that

undrained conditions are primarily controlled by the strength parameters not letting the volume change to occur. The Mohr-Coulomb model is demonstrated to be a logical first approxiamtion due to its simplicity. The models successfully capture the initial curved portion of the stress—strain relationship, reflecting the correct undrained stiffness mobilization that is critical for accurately predicting pore pressure development. This comparison underscores the importance of advanced constitutive modeling when analyzing undrained behavior, particularly for dynamic load applications, where the precise prediction of excess pore pressure generation and dissipation can significantly impact design decisions.

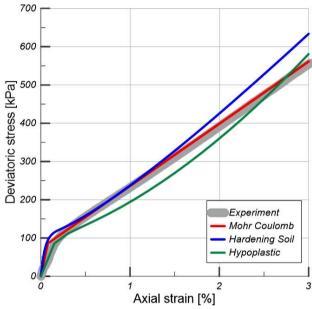


Figure 7. Comparison of undrained triaxial test results with different material models

From the above figures a qualitative assessment of the model capabilities can be given in a tabular way (Table 2) as follows:

Table 2. Qualitative assessment of the model capabilities

Soil behavior aspect	Mohr–Coulomb model	Hardening Soil model	Hypoplastic model
Nonlinear stress–strain behavior	poor	good	good
Stress-dependent stiffness	not captured	well captured	well captured
Small-strain behavior	poor	moderate	good
Computational efficiency	good	moderate	moderate
Parameter determination ease	good	moderate	hard

As can be seen from the Table 2, although both Hardening Soil and Hypoplastic models have similar good predictions on experimental values, the parameter determination can be the deciding factor in selection of the material model.

#### 6. CONCLUSIONS

This study presents a comprehensive comparison of numerical simulations for triaxial tests on Skopje sand using different constitutive models implemented in PLAXIS software. The results demonstrate that while the Mohr–Coulomb model adequately captures ultimate strength parameters, it significantly underperforms in reproducing the initial behavior of sand, including non-linear stress–strain relationships. The Hardening Soil model demonstrates markedly superior performance across multiple aspects of soil behavior, whereas the Hypoplastic model shows the best performance for the numerical simulations. Regarding the parameter determination, on the other hand, it might be time consuming using the hypoplastic material models.

These insights contribute to the broader understanding of soil behavior modeling and highlight the importance of selecting constitutive models that appropriately balance computational efficiency with predictive accuracy for specific geotechnical engineering applications.

# **APPENDIX**

```
Python input for Hypoplastic model
sand props = [
  "Identification", "Skopje Sand",
  "SoilModel", "userdefined",
  "gammaUnsat", 17, "gammaSat", 20,
  "PermHorizontalPrimary", 1e-5]
mat = g i.soilmat(*sand props)
# Set the DLL file and model
g i.setproperties(mat, "DLLfile", "udsm hps64.dll")
g i.setproperties(mat, "ModelinDLL", "Hypoplas.-sand")
# Set the Hypoplastic parameters using values from the table
g i.setproperties(mat, "User1", 35) # φc: Angle of internal friction [°]
g i.setproperties(mat, "User2", 1)
                                      # p<sub>t</sub> [kN/m<sup>2</sup>] (reference pressure, typical value)
q i.setproperties(mat, "User3", 2650000) #hs: Granulate hardness [kPa] (2650 MPa = 2650000 kPa)
g i.setproperties(mat, "User4", 0.39)
                                         # n: Exponent [-]
g i.setproperties(mat, "User5", 0.61)
                                         # eao: Minimum void ratio [-]
g_i.setproperties(mat, "User6", 0.98)
                                         # ec<sub>0</sub>: Critical void ratio [-]
g i.setproperties(mat, "User7", 1.09)
                                         # e<sub>10</sub>: max void ratio [-]
g i.setproperties(mat. "User8", 0.07)
                                         # α: Numerical parameter [-]
                                        #β: Numerical parameter [-]
g i.setproperties(mat, "User9", 2.0)
g i.setproperties(mat, "User16", 0.77)
                                        # SV: e<sub>0</sub> or e [-] ()
Python input for Mohr–Coulomb model
# Define material
sand props = [
  "Identification", " Skopje Sand ",
  "SoilModel", "Mohr-Coulomb",
  "gammaUnsat", 17,
                            # Unit weight above phreatic level [kN/m³]
  "gammaSat", 20.
                           # Unit weight below phreatic level [kN/m³]
  "Eref", 270000,
                           # Young's modulus [kN/m²]
  "nu", 0.2,
                      # Poisson's ratio [-]
  "cRef", 5,
                     # Cohesion [kN/m²]
  "phi", 35,
                     # Friction angle [°]
  "psi", 7,
                     # Dilatancy angle [°]
  "PermHorizontalPrimary", 1e-5 ]
```

```
Python input for Hardening Soil model #Define material sand_props = [ "Identification", "Skopje_Sand ", "SoilModel", "Hardening Soil", "gammaUnsat", 17, "gammaSat", 20, "E50Ref", 90000, "EoedRef", 90000, "EurRef", 270000, "CRef", 5, "phi", 35, "psi", 7, "PowerM", 0.5, "pRef", 100, "PermHorizontalPrimary", 1e-5]
```

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