doi.org/10.62683/SINARG2025.091

Research paper

SEISMIC DESIGN APROACH FOR TORSIONAL EFECTS ON RC DUCTILE BUILDINGS

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Abstract

This study investigates the influence of torsional effects on the seismic performance of reinforced concrete (RC) ductile buildings and provides a framework for their analysis and design. Key considerations include the relative positions of the center of mass (CM) and center of stiffness (CS), stiffness distribution, and the control of lateral displacements and inelastic rotations. Equations for determining total forces, torsional stiffness, radius of rotation, and system displacements under eccentric seismic loading are presented, along with methods to constrain inelastic twist in asymmetrical systems. The research adopts Paulay's classification of torsional systems into torsionally unrestrained and restrained categories, highlighting design strategies to limit torsional rotations while maintaining ductility. A numerical example illustrates the application of these concepts, demonstrating the design of a system with limited twist through optimal placement of vertical walls and consideration of ductility capacities. The findings emphasize the critical role of torsional considerations in modern seismic design, providing engineers with analytical tools to enhance structural resilience and prevent excessive damage during strong earthquakes.

Key words: *Ductility, Stiffness, Strength, Capacity, Torsional, Displacement, Inelastic, Response*

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1. INTRODUCTION

Structures with an asymmetric distribution of stiffness and strength in plan experience combined torsional and translational seismic responses (shown in Figure 1).

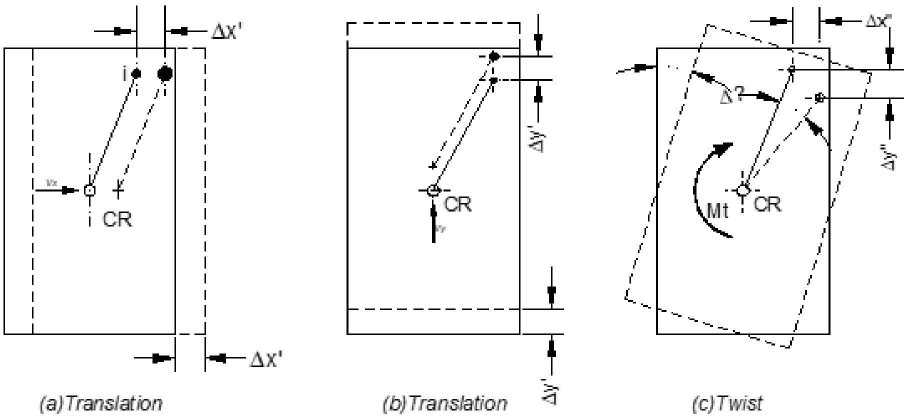

Figure 1. Combined torsional and translational responses

Plan irregularities often arise from non-uniform mass distribution, causing rotational motions of the floor slab even in systems where stiffness and strength are symmetric. A crucial step during the conceptual design phase is selecting an appropriate stiffness distribution for the structure. In systems expected to behave in a ductile manner, seismic design must consider the influence of torsional effects on inelastic deformations. Although this consideration is strongly recommended, it remains optional in most current design codes. However, recent developments emphasize their role in improving structural performance and reducing damage during strong earthquakes [1].

2. SEISMIC DESIGN CONSIDERATIONS ON THE DUCTILE BEHAVIOR OF STRUCTURES

In reinforced concrete (RC) ductile buildings, torsional effects arise due to irregularities in mass, stiffness, or geometry. In most building models, seismic forces are considered to act at the center of mass (CM) of the floor, while the lateral resistance is provided at the center of rigidity (CR), which is determined by the configuration and stiffness of the lateral force resisting elements. When the CM and CR do not coincide, eccentricity arises, generating torsional moments in addition to translational demands.

Figure 2 illustrates two plan views of a building subjected to lateral seismic forces:

- In (a), the Center of Rigidity (C_R) and Center of Mass (C_M) are offset, causing torsional rotation under earthquake loading (E), resulting in unequal lateral displacements (Δ) across the width.
- In (b), the Center of Base Shear (C_B) and Center of Mass (C_M) are aligned, resulting in pure translation without torsion.

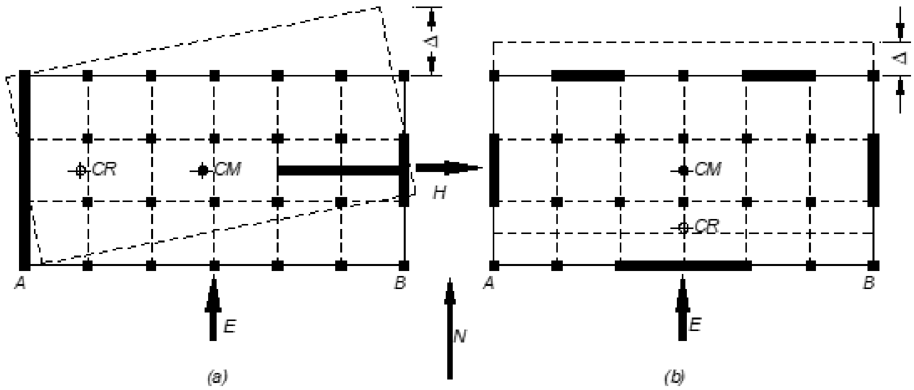

Figure 2. Typical distribution of vertical elements resisting lateral forces

This torsional response can significantly amplify the displacement and damage in certain structural elements, especially those located at the perimeter of the building. Seismic design must address these effects by ensuring that torsion-induced displacements remain within the ductility limits of structural elements. Key strategies include balancing stiffness distribution, controlling lateral displacements at points far from the center of stiffness, and detailing elements to accommodate inelastic torsional deformations. Since most structures are expected to respond inelastically, their design relies on ductile behavior characterized by controlled displacements and rotations. Assessing torsional effects during design is important to avoid unexpected failures. Including torsion in analysis helps ensure all critical elements perform safely under earthquakes [2].

For structures prone to torsional effects, the following performance criteria are generally adopted:

- Earthquake-induced deformations, including twisting, must not exceed the ductility capacity of structural elements.
- Maximum displacements, especially at points farthest from the C_S, should remain within specified limits, typically expressed as a percentage of the story height.

These provisions acknowledge the role of ductility in accommodating controlled inelastic deformations while maintaining overall stability and integrity. By ensuring displacement limits are respected, seismic design allows structures to withstand both translational and torsional demands without incurring excessive damage [3].

2.1. Determining Total Force and Torsional Stiffness in Structural Systems

In the analysis of a simplified structure with a few structural walls the forces acting on each element are divided into translational components and torque-related components [4].

The total force acting on an element in the x and y directions is the sum of the translational and torque-induced components:

$$V_{ix} = V_{ix}^{'} + V_{ix}^{''} \qquad V_{iy} = V_{iy}^{'} + V_{iy}^{''} \qquad (1a,b)$$

The translational force components V'_{ix} and V'_{iy} are calculated as:

$$V'_{ix} = (k_{ix}/\sum k_{ix})V_{Ex} \qquad V'_{iy} = (k_{iy}/\sum k_{iy})V_{Ey} \tag{2a,b}$$

Where: V_{Ex} and V_{Ey} are the external forces acting in the x and y directions.

The forces due to torque V_{ix}", V_{iy}" are defined by:

$$V''_{ix} = y_i k_{ix} M_t/K_t \qquad V''_{iy} = x_i k_{iy} M_t/K_t \tag{3a,b}$$

Where: Mt is the torque, and K_t is the torsional stiffness, defined as:

$$K_t = y_i^2 k_{ix} + x_i^2 k_{iy} \tag{4}$$

Where, x_i and y_i are the distances of the element from the center of twist and the k_{ix} and k_{iy} are the stiffness values in the x and y directions, respectively.

The radius of rotation relating to the torsional stiffness of the system can be defined as:

$$r_{kx}^2 = K_t/\sum k_{ix} \qquad r_{ky}^2 = K_t/\sum k_{iy} \tag{5a,b}$$

Where: $\sum k_{ix}$ and $\sum k_{iy}$, are the sum of the stiffness values of all elements in the structure.

2.2. Effect of Eccentricity on Displacement and Stiffness in Seismic Design

The displacement of structure Δ_m is defined by:

$$\Delta_m = \Delta'_m + \Delta''_m = V/\sum k_i + \theta_t e_s \tag{6}$$

Where: Δ'_m and Δ''_m are the translational and rotational components of displacement of mass and θ_t is the angle of twist, defined as:

$$\theta_t = M_t/K_t = V\, e_r\, /K_t \tag{7}$$

With these equations, the displacement of structure Δ_m can be established by the equation:

$$\Delta_m = V[1/\sum k_i + \ e_r^2/K_t] \tag{8}$$

Then, the stiffness of the system K_S is determined by the equation:

$$K_s = V/\Delta_m = \sum k_i\, /(1 + \ e_r^2/K_t) \tag{9}$$

It shows that, whenever the eccentricity of inertia exists, the inertia of the system is smaller than the sum of the weights of the elements. Therefore, the displacement at the yield point of the system is:

$$\Delta_y = V_E/K_s \tag{10}$$

2.3. Constrain against Twisting

An important aspect of the inelastic behaviour of an asymmetrical structural system is the consideration of the level of control over the inelastic twist. However, one of the design goals is to constrain the system against unlimited inelastic twist. The system in this way, during the development of the translational displacement expected for the ductility capacity, μ_Δ, preserves a certain amount of stiffness in the twist and is determined with the relation:

$$K_{tr} = \lambda_t K_t \tag{11}$$

Where: $\lambda_t \leq 1.0$

The parameter λt, in equation (11), expresses the degree of restriction of the twist:

$$\lambda_{tx} = \sum (x_i^2 k_{iy})/K_t \qquad \lambda_{ty} = \sum (y_i^2 k_{ix})/K_t \qquad (12a,b)$$

In the ultimate limit state, the twisting moment that must be accepted can be determined by the equation:

$$M_{tu} = e_v V_E \qquad (14)$$

Where: e_V is the eccentricity, and it should satisfy $e_v \leq e_r$, where e_r is the design (or strength-based) eccentricity.

These equations and classifications provide a framework for analysing and designing ductile structures about torsional response, considering factors such as stiffness, displacements, and restriction of twist.

3. A CLASSIFICATION OF TORSIONAL DUCTILE SYSTEMS

The classification of torsional systems proposed by Paulay differentiates between torsionally unrestrained and torsionally restrained systems based on their ability to control torsional rotation during seismic activity in the principal directions of a building These classifications are particularly important for ductile structures, which are designed to withstand seismic forces. Thus, according to Paulay, the systems of ductile structures for torsional response are classified into two categories:

1. Torsional unrestrained systems with unlimited twist, where λt =0 and
2. Torsional restrained systems with limited twist, where λt >0.15.

3.1. Torsional Unrestrained Systems

Systems with λ_t values above this threshold are deemed to have sufficient torsional resistance to qualify as torsionally restrained systems. The simplest structure (Paualy) (shown in Figure 3) with transverse elements serves as an example of a torsionally unrestrained system.

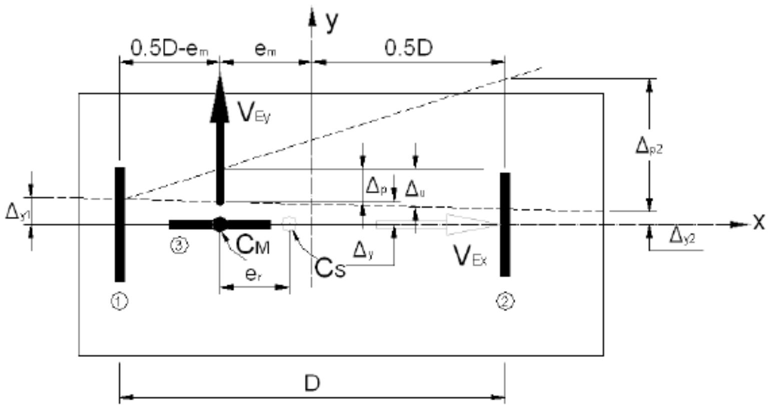

Figure 3. Characteristics of torsional unrestrained system

The yield displacement of the system which considers the contributions of both elements will be:

$$\Delta_y = (V_1/k_1)(0{,}5 + e_m/D) + (V_2/k_2)\,(0{,}5 - e_m/D) \quad (15)$$

The displacement ductility applicable to the system can be defined by the expression given by Paulay:

$$\mu_\Delta = \Delta_u/\Delta_y = 1 + \Delta_p/\Delta_y = [(\mu_{max} - 1)/(1 + \omega_0)] + 1 \le \mu_{max} \quad (16)$$

Where:

$$\omega_0 = [V_1 k_2\,(0{,}5 + e_m/D)]/[V_2 k_1 (0{,}5 - e_m/D)] \quad (17)$$

Therefore, the maximum necessary requirement of the system ductility should be limited, and it may be shown that (Eq.17) is also valid when more than two elements are positioned to resist the force V_{Ey}. The force V_{Ey} can be established by the equilibrium of the cutting force components, $V_{Ey} = V_1 + V_2$.

3.2. Torsional Restrained Systems

The value λ_t > 0.15 represents a threshold limit proposed by Paulay, indicating the minimum level of torsional restraint required for a system to be considered torsionally restrained (shown in Figure 4).

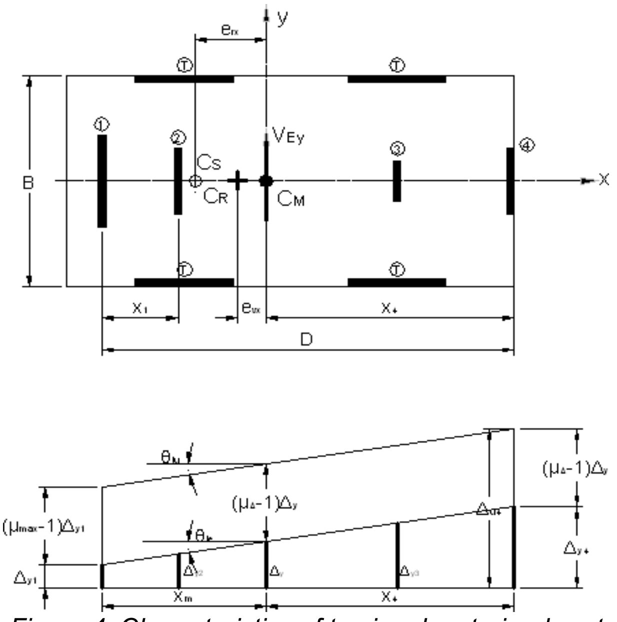

Figure 4. Characteristics of torsional restrained system

The maximum acceptable requirement of ductile displacement of the system is limited:

$$\mu_\Delta = [(\mu_{max} - 1)/(1 + \omega_{tr})] + 1 \quad (18)$$

Where: the factor ω, applicable to strained torsional systems:

$$\omega_{tr} = [V_{Ey} k_1/V_1 K_{sy}] - 1 \quad (19)$$

Overall, this classification and the associated equations provide a framework for designing ductile structures with a focus on limiting torsional rotations and ensuring the inelastic response of the system remains within acceptable limits [5].

4. DETERMINATION OF LIMITED TORSION, CAPACITY AND DISPLACEMENT IN DUCTILE SYSTEMS

In the numerical example, the torsional response of ductile systems is analyzed. This principle is based on the deformation capacity and the corresponding demand of the system, especially of its critical elements, more precisely on the torsional force. The analysis determines the inelastic displacements of the elements depending on the inelastic motion of the system [4].

On this basis, the critical elements are provided with lateral force resistance, especially in the length of the plane, against the demand of inelastic deformations. For the determination of the displacements, the ductility factor is used for known displacements. By focusing on controlling the inelastic displacements of the elements, possible torsional effects are not considered by the distribution of masses, which affects the instantaneous position of the center of mass [6].

4.1. Numerical Example

In the provided example, the proposed steps for designing a system with limited twist are illustrated using (Figure 5). The system in the example meets the criteria for limited twisting, which is important in building design. The figure shows a shaded area covering 16 floors and an unshaded area with 3 floors. The relative stiffness values (ki) were chosen to limit twisting in the top 13 floors and the whole structure. Vertical walls are used to resist lateral forces.

The results focus on the 1st and 4th floors, assuming all elements have a displacement ductility capacity of μ_{max}=5. This example emphasizes the importance of limiting twist through the smart placement of vertical walls and considering ductility in the design.

4.1.1. Input Data

The input data considered in this research paper are summarized in the form of table. The data are given in the direction of axes x and y of the building.

Table 1. General input data of the considered building

Direction:	x	y
Building floor plane: [m]	L_X=24	L_y=24
Walls lengths: [m]	l_{4x}=5, l_{5x}=6.3, l_{6x}=5	l_{1y}=5, l_{2y}=6.3, l_{3y}=5
Wall thickness: [mm]	d=200	
Module of the building: [m]	B=6.0	
Moment of inertia: [m^4]	I_{4x}=2.08, I_{5x}=4.16, I_{6x}=2.08	I_{1y}=2.08, I_{2y}=4.16, I_{3y}=2.08
Stiffness: [m^4]	k_{4x}=2.08, k_{5x}=4.16, k_{6x}=2.08	k_{1y}=2.08, k_{2y}=4.16,k_{3y}=2.08

4.1.2. Input Data for the First Floor

The positions of the centers C_M and C_S and the corresponding eccentricities are shown as a function of the horizontal platform dimension B=6.0 m in (Figure 5b).

Table 2. Input data for the first floor

Direction:	x	y
Centre of Mass:	$x_M = 1.422$	$y_M = -3.726$
Stiffness:	$\sum k_{ix} = 8.32$	$\sum k_{iy} = 8.32$
Centre of Stiffness:	$x_s = 12$	$y_s = 12$
Torsional stiffness:	$K_t = 1198.08$	
Radius of rotation:	$r_{kx} = 12$	$r_{ky} = 12$
Ecctrencity:	$e_{rx} = 1.422$	$e_{ry} = 3.726$
Troques:	$M_{tx} = 3.726V_{Ex}$	$M_{ty} = 1.422V_{Ey}$
Parameter of degree of torsional restraint:	$\lambda_{ix} = 0.5$	$\lambda_{iy} = 0.5$
Strengths of Elements:	$V_{4x} = 0.328V_{Ex}$ $V_{5x} = 0.5V_{Ex}$ $V_{6x} = 0.172V_{Ex}$	$V_{1y} = 0.22V_{Ey}$ $V_{3y} = 0.28V_{Ey}$ $V_{2y} = 0.5V_{Ey}$
The smallest force for the walls:	$V_{6x} = 0.172V_{Ex}$	$V_{1y} = 0.22V_{Ey}$

4.1.3. Input Data for the Fourth Floor

The positions of the centers C_M and C_S and the corresponding eccentricities are shown as a function of the horizontal platform dimension B=6.0 m in (Figure 5c).

Table 3. Input data for the fourth floor

Direction:	x	y
Centre of Mass:	$x_M = -1.0$	$y_M = -0.8$
Stiffness:	$\sum k_{ix} = 6.24$	$\sum k_{iy} = 6.24$
Centre of Stiffness:	$x_s = 10$	$y_s = 8$
Torsional stiffness:	$K_t = 399.36$	
Radius of rotation:	$r_{kx} = 6.93$	$r_{ky} = 6.93$
Ecctrencity:	$e_{rx} = 1.0$	$e_{ry} = 0.8$
Troques:	$M_{tx} = 0.8\ V_{Ex}$	$M_{ty} = 1.0\ V_{Ey}$
Strengths of Elements:	$V_{4x} = 0.366V_{Ex}$ $V_{5x} = 0.634V_{Ex}$	$V_{2y} = 0.709V_{Ey}$ $V_{3y} = 0.291V_{Ey}$
The smallest force for the walls:	$V_{4x} = 0.366V_{Ex}$	$V_{3y} = 0.291V_{Ey}$

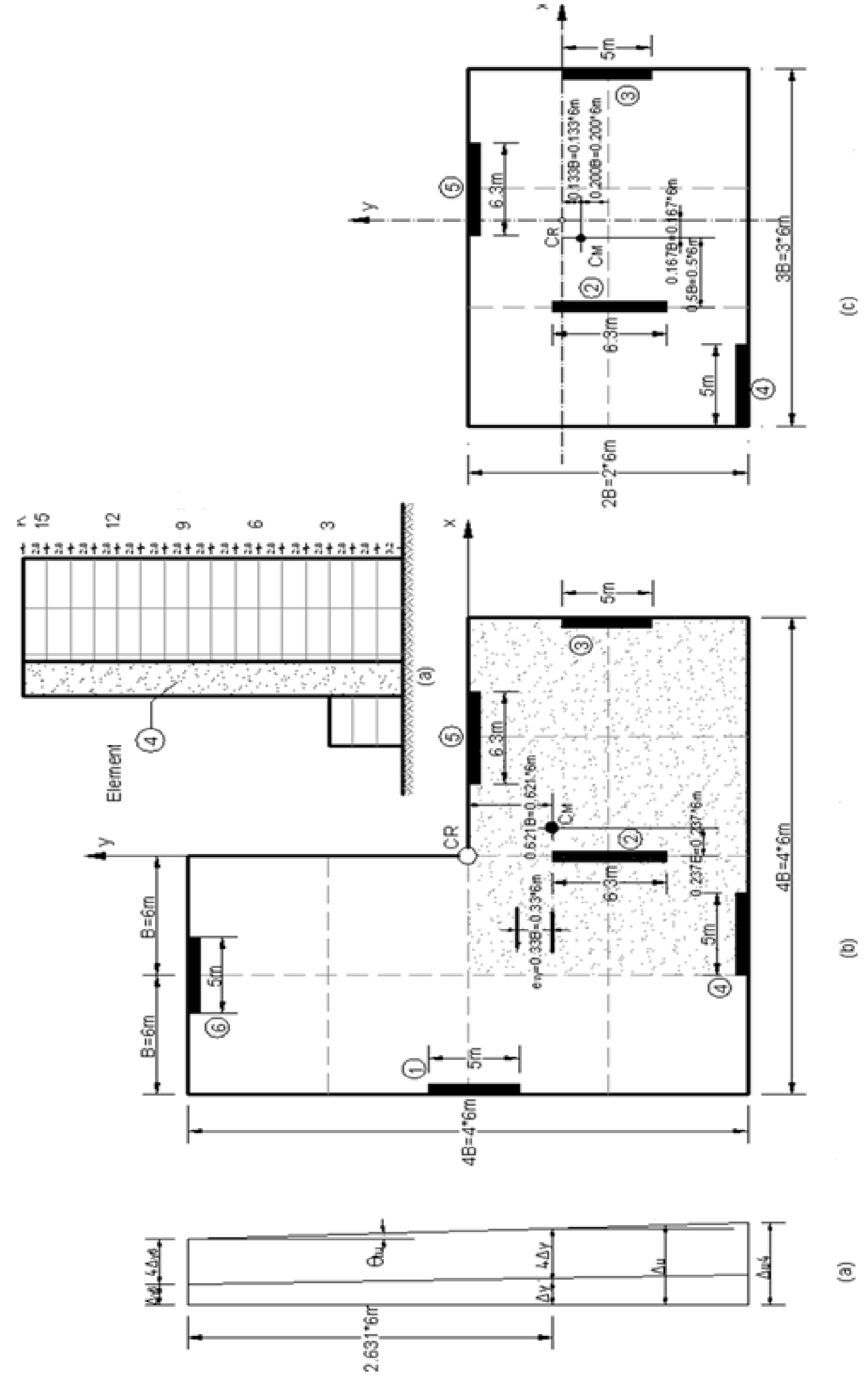

Figure 5. Example of torsionally restrained building, a) displacements,
b,c) layout of the first and fourth floor (adapted from reference: [3])

5. DISCUSSIONS

The numerical example highlights the effectiveness of torsional restraint in seismic design through a focused analysis of a ductile structural system. By strategically distributing stiffness and using vertical walls to resist lateral forces, the design meets the criteria for limited torsional response, particularly on the 1st and 4th floors.

The results demonstrate that controlling inelastic displacements and ensuring the ductility of critical elements especially walls with lower yield strength is crucial to achieving seismic resilience. In both examined floors, eccentricities between the center of mass and the center of stiffness introduced torsional moments, yet the resulting displacements remained within acceptable ductility limits ($\mu_{\Delta x}$ and $\mu_{\Delta y}$ below the capacity μ_{max}=5). This indicates that the system can undergo significant inelastic deformation without compromising structural integrity.

Moreover, the analysis shows that the placement and stiffness of structural walls directly influence torsional stiffness (K_t), radius of rotation (r_k), and eccentricity-induced torques (M_t). The values of the torsional restraint parameters (λ_i and ω_{rt}) further confirm that the system is effectively restrained against excessive twist.

In summary, the example validates the importance of:

- Optimizing wall locations and stiffness distribution.
- Evaluating torsional effects through eccentricity and torque parameters.
- Ensuring that critical elements have sufficient ductility.
- Controlling inelastic displacements to avoid structural failure.

This design approach ensures seismic performance while reducing the risk of torsional amplification, particularly in irregular structures or those with asymmetric mass and stiffness distribution.

6. CONCLUSIONS

Torsion is a critical factor in the seismic design of buildings, particularly in ductile systems intended to withstand significant lateral forces. It refers to the twisting or rotational response of a structure when subjected to seismic-induced moments, which can lead to uneven displacement demands on structural elements.

- For the first floor, seismic analysis identifies walls 1 and 6 as the critical elements due to their relatively low yield strength. The combined stiffness in the x- and y-directions, along with torsional factors (ω_{rtx} = 0.593, ω_{rty} = 0.154), indicates moderate torsional restraint. The calculated displacement ductility factors ($\mu_{\Delta x}$ = 3.51, $\mu_{\Delta y}$ = 4.47) confirm that the system can accommodate inelastic deformations within acceptable limits, maintaining controlled torsional behavior.
- For the fourth floor, walls 3 and 5 are identified as critical due to their lower yield strength. The measured stiffness values (K_{sx} = 6.158, K_{sy} = 6.115) and torsional factors (ω_{rtx} = 0.062, ω_{rty} = 0.167) reflect adequate torsional restraint. Displacement ductility factors ($\mu_{\Delta x}$ = 4.77, $\mu_{\Delta y}$ = 4.43) demonstrate that the system can sustain significant inelastic deformations while ensuring a stable torsional response.

Overall, the results highlight the importance of strategically locating and sizing critical walls, combined with explicit consideration of torsional effects, to maintain structural performance and safety under seismic loading.

In summary, considering the torsion effect in the design of buildings is essential for ductile systems intended to withstand seismic activities. Designers should account for the inelastic displacement caused by torsion in certain elements and take measures to limit the effects of unlimited inelastic torsion.

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